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TWO-DOUBLET HIGGS STRUCTURE AT THE ELECTROWEAK ENERGY SCALE*

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ABSTRACT

The existence of supersymmetry above a few TeV and that of two Higgs doublets at the electroweak energy scale do not necessarily result in the minimal supersymmetric standard model (MSSM). An interesting counter example is given with $m_h < \sqrt{2} m_W$ + radiative corrections $\sim$ 145 GeV instead of $m_h < m_Z$ + radiative corrections $\sim$ 128 GeV in the MSSM.

1. Preamble

This year, there have been two very reassuring events in high-energy physics. One is the possible discovery of the top quark at Fermilab;[1] and the other is the continuation of this series of Adriatic Meetings on Particle Physics, now the 7th and for the first time in Brijuni, Croatia. The Chairman Prof. Tadic and all the other organizers are to be congratulated.

2. Introduction

The reported central value of the top-quark mass is 174 GeV which is exactly equal to the vacuum expectation value v of electroweak symmetry breaking given by

$$\frac{G_F}{\sqrt{2}} = \frac{1}{4v^2}. \tag{1}$$

This means that the era of experimental exploration of physics at the electroweak energy scale has begun. What is the next particle to be discovered? In the standard model, there is just the one Higgs boson, but in most of its extensions, there are likely to be two Higgs doublets.[2] In the very popular minimal supersymmetric standard

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model (MSSM), two Higgs doublets are in fact required. The lightest neutral scalar boson must also satisfy the mass bound

$$m_h < m_Z + \text{radiative corrections}, \tag{2}$$

which is about 128 GeV for $m_t = 174$ GeV. This bound is saturated in the limit of a large pseudoscalar mass (say a few TeV), where the two-doublet Higgs structure of the MSSM reduces to only one Higgs boson at the electroweak energy scale as in the standard model.

The implicit assumption of the MSSM is that at the energy scale of soft supersymmetry breaking, say a few TeV, the gauge group is the standard SU(3)×SU(2)×U(1). If the latter is something larger but it breaks down to the standard gauge group also at a few TeV, then the structure of the Higgs potential is determined by the scalar particle content needed for that symmetry breaking. Furthermore, the quartic scalar couplings are related to the gauge couplings of the larger theory as well as other couplings appearing in its superpotential.

At the electroweak energy scale, the reduced Higgs potential may contain only two scalar doublets, but their quartic couplings are generally not those of the MSSM. In this talk I will describe one such counter example[3] based on a very interesting supersymmetric left-right model proposed some years ago.[4] In my second talk[5] I will give more details of that model itself and describe some recent results on its possible unification at the 10^{16} GeV energy scale, as well as its effect on the precision measurements of $Z \to$ leptons.

3. The Two-Doublet Higgs Potential

Consider two Higgs doublets $\Phi_{1,2} = (\phi^+_{1,2}, \phi^0_{1,2})$ and the Higgs potential

$$\begin{aligned} V &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ &+ \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ &+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \frac{1}{2}\lambda_5^* (\Phi_2^\dagger \Phi_1)^2. \end{aligned} \tag{3}$$

In the MSSM, there are the well-known constraints

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2), \quad \lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2, \quad \lambda_4 = -\frac{1}{2}g_2^2, \quad \lambda_5 = 0, \tag{4}$$

where g_1 and g_2 are the U(1) and SU(2) gauge couplings of the standard model respectively. As $\phi^0_{1,2}$ acquire vacuum expectation values $v_{1,2}$, two tree-level sum rules are obtained:

$$m^2_{h^0} + m^2_{H^0} = m_Z^2 + m_A^2, \quad m^2_{H^\pm} = m_W^2 + m_A^2, \tag{5}$$

where the pseudoscalar mass m_A is given by

$$m_A^2 = -\mu_{12}^2 (\tan\beta + \cot\beta), \quad \tan\beta \equiv v_2/v_1. \tag{6}$$

Note that only the gauge couplings contribute to the λ's. This is because that with only two $\mathrm{SU}(2) \times \mathrm{U}(1)$ Higgs superfields, there is no cubic invariant in the superpotential and thus no additional coupling.

4. The E_6-Inspired Left-Right Model

Consider now the gauge group $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)$ but with an unconventional assignment of fermions[4] including an exotic quark h of electric charge $-1/3$.

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \sim (2,1,1/6), \qquad d_R \sim (1,1,-1/3), \tag{7}$$

$$\begin{pmatrix} u \\ h \end{pmatrix}_R \sim (1,2,1/6), \qquad h_L \sim (1,1,-1/3), \tag{8}$$

$$\Phi_1 \equiv \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \sim (2,1,1/2), \qquad \chi \equiv \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix} \sim (1,2,1/2), \tag{9}$$

$$\eta \equiv \begin{pmatrix} \overline{\phi_2^0} & \eta^+ \\ -\phi_2^- & \eta^0 \end{pmatrix} \sim (2,2,0). \tag{10}$$

Note that the mass matrices for the u, d, and h quarks are proportional to different vacuum expectation values, *i.e.* $\langle\phi_2^0\rangle$, $\langle\phi_1^0\rangle$, and $\langle\chi^0\rangle$ respectively. Hence flavor-changing neutral currents are guaranteed to be absent at tree level in this model, as opposed to the conventional left-right model where they are unavoidable. Note also that $\Phi_1^\dagger \tilde{\eta} \chi$ is now an allowed term in the superpotential, where $\tilde{\eta} \equiv \sigma_2 \eta^* \sigma_2$, so that its coupling f also contributes to the quartic scalar couplings of this model's Higgs potential.

Let G_1 be the U(1) gauge coupling and G_2 the coupling of both SU(2)'s. Then

$$\begin{aligned} V &= V_{soft} + \frac{1}{8}(G_1^2+G_2^2)[(\Phi_1^\dagger\Phi_1)^2 + (\chi^\dagger\chi)^2] \\ &+ \frac{1}{4}G_2^2[(\mathrm{Tr}\eta^\dagger\eta)^2 - (\mathrm{Tr}\eta^\dagger\tilde{\eta})(\mathrm{Tr}\tilde{\eta}^\dagger\eta)] + (f^2 - \frac{1}{4}G_2^2)(\Phi_1^\dagger\Phi_1 + \chi^\dagger\chi)\mathrm{Tr}\eta^\dagger\eta \\ &- (f^2 - \frac{1}{2}G_2^2)(\Phi_1^\dagger\eta\eta^\dagger\Phi_1 + \chi^\dagger\eta^\dagger\eta\chi) + (f^2 - \frac{1}{4}G_1^2)(\Phi_1^\dagger\Phi_1)(\chi^\dagger\chi), \end{aligned} \tag{11}$$

where V_{soft} contains terms of dimensions 2 and 3, and breaks the supersymmetry. Let χ^0 acquire a vacuum expectation value $u \neq 0$. Then $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)$ breaks down to the standard $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ with $m^2(\sqrt{2}Re\chi^0) = (G_1^2 + G_2^2)u^2/2$ and $m^2(\eta^+, \eta^0) = G_2^2 u^2/2$. These heavy particles can be integrated out at the electroweak energy scale where only $\Phi_{1,2}$ are left.

5. Reduced Higgs Potential of the Left-Right Model

The quartic scalar couplings of the reduced Higgs potential at the electroweak

energy scale are now given by

$$\lambda_1 = \frac{1}{4}(G_1^2 + G_2^2) - \frac{(4f^2 - G_1^2)^2}{4(G_1^2 + G_2^2)}, \tag{12}$$

$$\lambda_2 = \frac{1}{2}G_2^2 - \frac{(4f^2 - G_2^2)^2}{4(G_1^2 + G_2^2)}, \tag{13}$$

$$\lambda_3 = \frac{1}{4}G_2^2 - \frac{(4f^2 - G_1^2)(4f^2 - G_2^2)}{4(G_1^2 + G_2^2)}, \tag{14}$$

$$\lambda_4 = f^2 - \frac{1}{2}G_2^2, \quad \lambda_5 = 0, \tag{15}$$

where the second terms on the right-hand sides of the equations for $\lambda_{1,2,3}$ come from the cubic interactions of $\sqrt{2}Re\chi^0$ which are proportional to u. When divided by the square of its mass, these contributions do not vanish even if u becomes very large. This is another example of nondecoupling.

In the limit $f = 0$ and using the tree-level boundary conditions

$$G_2 = g_2, \quad G_1^{-2} + G_2^{-2} = g_1^{-2}, \tag{16}$$

it can easily be shown from the above that the MSSM is recovered. However, f is in general nonzero, although it does have an upper bound because V must be bounded from below. Hence

$$0 \leq f^2 \leq \frac{1}{4}(g_1^2 + g_2^2)\left(1 - \frac{g_1^2}{g_2^2}\right)^{-1}, \tag{17}$$

where the maximum value is obtained if V_{soft} is aslo left-right symmetric. For illustration, let $f = f_{max}$ and $x \equiv \sin^2\theta_W$, then

$$\lambda_1 = 0, \quad \lambda_2 = \frac{e^2}{2x}\left[1 - \frac{2x^2}{(1-x)(1-2x)}\right], \tag{18}$$

$$\lambda_3 = \frac{e^2}{4x}\left[1 - \frac{2x}{1-2x}\right] = -\lambda_4, \quad \lambda_5 = 0. \tag{19}$$

It is clear that the MSSM conditions of Eq. (4) are no longer appropriate.

6. Phenomenological Consequences

At tree level, the 2×2 mass-squared matrix of the two neutral Higgs scalar bosons of the two-doublet model is given by

$$\mathcal{M}_{h,H}^2 = \left[\begin{array}{cc} 2\lambda_1 v_1^2 - \mu_{12}^2 v_2/v_1 & 2(\lambda_3+\lambda_4)v_1 v_2 + \mu_{12}^2 \\ 2(\lambda_3+\lambda_4)v_1 v_2 + \mu_{12}^2 & 2\lambda_2 v_2^2 - \mu_{12}^2 v_1/v_2 \end{array} \right]. \tag{20}$$

Hence there is an upper bound on the lighter of the two bosons:

$$m_h^2 < 2(v_1^2 + v_2^2)[\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + 2(\lambda_3+\lambda_4)\sin^2\beta\cos^2\beta] + \epsilon, \tag{21}$$

where $\tan\beta \equiv v_2/v_1$ as defined previously, and ϵ is a radiative correction due to the presence of the top quark:

$$\epsilon = \frac{3g_2^2 m_t^4}{8\pi^2 m_W^2} \ln\left(1 + \frac{\tilde{m}^2}{m_t^2}\right), \tag{22}$$

where $\tilde{m}$ is an effective mass of the two scalar top quarks. The MSSM bound is given by

$$m_h^2(f=0) < (m_Z \cos 2\beta)^2 + \epsilon, \tag{23}$$

whereas in this model with $f = f_{max}$, it is given by

$$m_h^2(f = f_{max}) < 2m_W^2 \left[1 - \frac{2\sin^4\theta_W}{\cos^2\theta_W \cos 2\theta_W}\right] \sin^4\beta + \epsilon. \tag{24}$$

However, m_h is maximized not at f_{max}, but at[6]

$$f_0^2 = \frac{1}{4} g_2^2 \left(1 - \cos^4\beta + \frac{g_1^2}{g_2^2}\cos 2\beta\right)\left(1 - \frac{g_1^2}{g_2^2}\right)^{-1} \tag{25}$$

which is less than f_{max}^2 for all β. The upper bounds on m_h for $f = f_0$, $f = f_{max}$, and $f = 0$ are plotted together in Fig. 1. The absolute upper bound on m_h is then given by

$$m_h^2 < 2m_W^2 + \epsilon, \tag{26}$$

which occurs at $\cos^2\beta = 0$. Using $m_t = 174$ GeV and $\tilde{m} = 1$ TeV, this implies that

$$m_h < 145 \text{ GeV} \tag{27}$$

in this model, as opposed to the upper bound of 128 GeV in the MSSM ($f = 0$).

7. Another Scenario

It has been shown in the above that the existence of a cubic term in the superpotential will change the two-doublet Higgs structure at the electroweak energy scale. This occurs naturally for a gauge symmetry larger than the standard SU(2) × U(1) because the superfields involved are likely to be precisely those required for the symmetry breaking and the generation of fermion masses, as is the case of the E_6-inspired supersymmetric left-right model.[4] However, even if the gauge symmetry is only SU(2) × U(1), a singlet scalar superfield N may be added so that the term $f\Phi_1^\dagger \Phi_2 N$ is allowed. Now $\lambda_{1,2,3,5}$ are as in the MSSM, but

$$\lambda_4 = -\frac{1}{2} g_2^2 + f^2. \tag{28}$$

Assume that $\langle N \rangle = 0$, then

$$m_{H^\pm}^2 = m_W^2 + m_A^2 - f^2(v_1^2 + v_2^2), \tag{29}$$

hence $m_{H^\pm} < m_W$ is now possible, in contrast to the MSSM or the left-right model discussed above. Note that λ_4 gets the extra contribution f^2, regardless of how heavy N is. This shows how sensitive the supersymmetric two-doublet Higgs structure is to the details of a possible larger theory.

Although the other sum rule of the MSSM [the first one in Eq. (5)] still holds in this scenario, the upper bound on m_h is different because of f. For a given value of m_A, this upper bound is maximized at $f^2(v_1^2 + v_2^2) = (m_A^2 + m_Z^2)/2$, making $\mathcal{M}^2_{h,H}$ of Eq. (18) diagonal. Hence

$$m_h^2 < m_Z^2 \cos^2\beta + m_A^2 \sin^2\beta, \tag{30}$$

or

$$m_h^2 < m_Z^2 \sin^2\beta + m_A^2 \cos^2\beta + \frac{\epsilon}{\sin^2\beta}, \tag{31}$$

whichever is smaller. Since the value of m_A is unrestricted, this means that there is no real upper bound on m_h. The lesson to be learned here is that with a minimal change in the larger theory, the two-doublet Higgs structure at the electroweak energy scale can be drastically different. In the above case of the addition of a seemingly harmless singlet, both of the well-known bounds of the MSSM on the masses of scalar bosons are removed.

8. Conclusions

(1) Even if supersymmetry exists and there are only two Higgs doublets at the electroweak energy scale, the minimal supersymmetric standard model (MSSM) is not the only possibility.

(2) For $m_t = 174$ GeV, $m_h > 128$ GeV rules out the MSSM, and $m_h > 145$ GeV rules out the E_6-inspired supersymmetric left-right model.

(3) If two Higgs doublets are discovered and their masses determined, it may even be possible to deduce what precisely the larger theory is at the scale of supersymmetry breaking.

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